

Maplet for Linear Fredholm Integral Equations of Second Kind (FIESK)

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Abstract—the main purpose of this paper is to propose a Maplet application of the solution methods, Neumann, Eigen function, Adomian Decomposition Method (ADM), and Simple Method for Linear Fredholm Integral Equations of Second kind. Furthermore, to make 2-D, 3-D graphing of solutions and how they can be used as effective educational tools for both students and instructors.

Index Terms—Linear Fredholm integral equation; Neumann Method; Eigen function Method; Adomian Decomposition Method; Maplet

I. INTRODUCTION

Integral equations have been one of the principal tools in various areas of science such as applied mathematics, physics, biology and engineering. On the other hand, integral equations are encountered in numerous applications in many fields including continuum mechanics, potential theory, geophysics, electricity and magnetism, kinetic theory of gases, hereditary phenomena in physics and biology, renewal theory, quantum mechanics, radiation, optimization, optimal control systems, communication theory, mathematical economics, population genetics, queuing theory, medicine, mathematical problems of radiative equilibrium, particle transport problems of astrophysics and reactor theory, acoustics, fluid mechanics, steady state heat conduction, fracture mechanics, and radiative heat transfer problems[1].

From few types of the integral equations, the most frequently investigated integral equations are Fredholm linear equations and its nonlinear counterpart. However, in this paper, linear Fredholm integral equations of the second kind are considered.

II. MOTIVATION

Generally, linear Fredholm integral equations of second kind can be defined as follows

$$u(x) = f(x) + \lambda \int_a^b K(x,t)u(t)dt \quad (1)$$

where the parameter λ , kernel $K(x,t)$ and free term f are given, and u is the unknown function to be determined.

There are some methods to obtain exact and approximate solutions of Linear Fredholm Integral Equations of second kind, such as Neumann series method [2], Eigen function method[3], Adomian Decomposition method(ADM)[4], and Simple Method[5]. In this work the application of these methods will be considered.

III. USING MAPLET

In recent years a significant number of interactive tools have been created to display mathematics over the internet using any browser. Two of most known software is web Mathematica (see [6, 7] and MapleNet see [6]). Educators engaging in the use of the internet in teaching Mathematics are using interactive tools to present and discover mathematical concept.

Virtualizing integral equation concepts enhance the ability of the student to comprehend the dynamic of the three dimensional mathematical concept. We have created an application with Maplets that contains animations for the concept covered in the integral equations course by keeping this philosophy in mind.

Maple and similar computer algebra systems (CAS) are changing the way we study, teach and apply mathematics. Each of those systems combines comprehensive collection mathematical functionalities, a graphical and computational environment, a programming language and many other features in one software package, making it possible to perform sophisticated mathematical computations with a single or a sequence of commands. Currently, Maple and Mathematica are arguably the two most successful computer algebra systems, along with Matlab that is mainly built for efficient numerical computations.

Maple is a product of Maple Soft, one of the leading providers of software tools for mathematical, engineering and scientific computing. It includes rich and user-friendly interfaces along with a large collection of functionalities for manipulating mathematical expressions and numbers. Furthermore, Maple comes with a simple programming

language that is capable of being used to implement sophisticated algorithms for scientific computing. It is an excellent platform for introducing scientific computing, basic programming, mathematical experimentation and simulation.

A Maplet consists of one or more windows which interact with the user by means of buttons, checkboxes, text fields, and other standard graphical controls. Maple is a powerful problem-solving environment capable of doing quite advanced mathematics, but the price of this power is considerable complexity. With a Maplet, this complexity can be retained but restrained: a simple menu interface, tailored to a specific task or problem [8].

The Maplets packages of Maple allow designing graphics interfaces with the capability to interact with the mathematical kernel. With this tool we have developed a Maplet application of linear Fredholm integral equations of second kind. The Maplets are generated basically using Maple code and then displayed over the web as JSP application through MapleNet. The use of Maple through MapleNet gives the user with access to the internet using any browser the interactive capabilities of the animations.

The advantage of using interactive tools such as the Maplets is that it takes away the somehow complicated code to investigate the concept. The Maplet as well as other similar web applications allow the user to enter a minimum codify information. The main goal of the application is for the user to experiment and discover mathematical concept, develop the intuition and make conjectures.

IV. MAPLET OF LINEAR FREDHOLM INTEGRAL EQUATIONS OF SECOND KIND

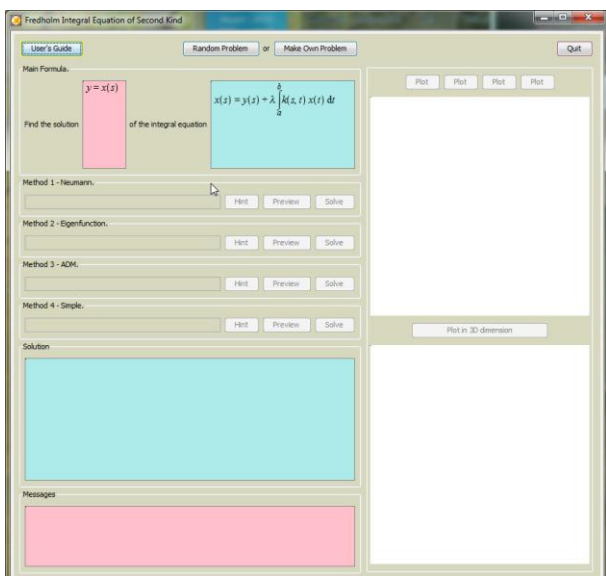


Fig. 1. Main display of the Maplet

Figure 1. The main display of the Maplet. There are different panels with buttons on the main display of the

Maples, as solution methods, methods panel, answer panel, message panel, 2-D graph panel and 3-D graph panel. The user can control all features of the Maplet. User can enter the inputs of the linear Fredholm integral equations. It is possible to create random examples or users own problem.

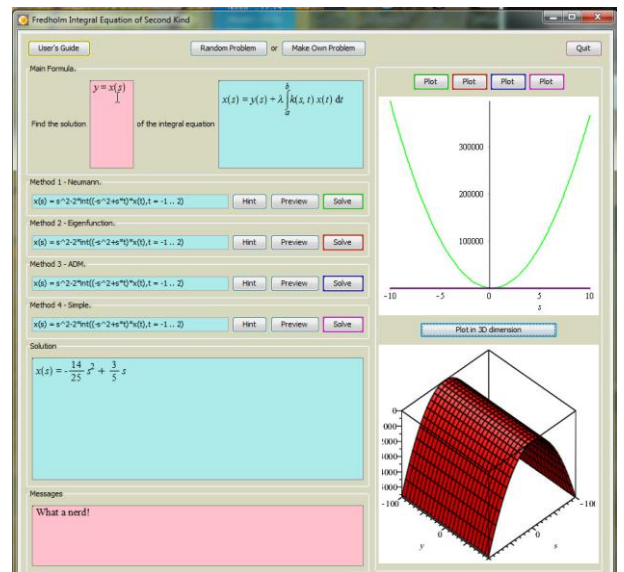


Fig. 2 .Main window of the Maplet with an Example

Figure 2 is an example of creating random problem. In the figure one can see the answer by using different methods, a message to user about solution and using maplet, 2-D graphics , 3-D graphics.

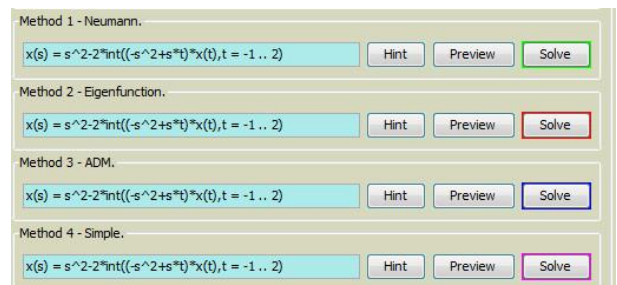


Fig. 3 .Solution methods pane

Figure 3 is the panel of solution methods of linear Fredholm integral equations. There can be seen the answers using the methods of Neumann, Eigen function, ADM and simple. There are also “Hint”, “Preview” and “Solve” buttons on the panel. The Hint button gives some information about the used method. The “Preview” button displays the mathematical view of the answer and the “Solve” button is solved the equation.

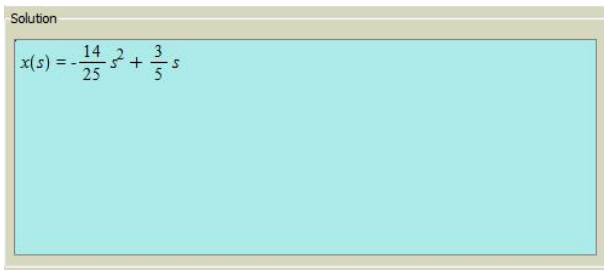


Fig. 4 .Solution panel of the Maplet

Figure 4 is the solution panel of the entered integral equation. In the panel user can see the answer of each used method. Only one result can be seen on the panel.



Fig. 5 .Main menu panel

Figure 5 is the main menu panel. There are “User’s guide”, “Random Problem”, and ”Make Own Problem”. User’s guide panel gives some hints about the use of Maplet. Random Problem creates random problems of the form linear Fredholm integral equation. User can create his/her own problem by using Make Own Problem button.

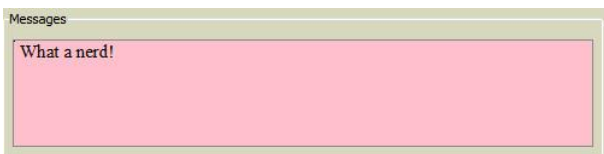


Fig. 6 .The message panel of the Maplet

Figure 6 is the message panel of the Maplet. In this panel Maplet interacts with the user. Messages are motivational for the user.

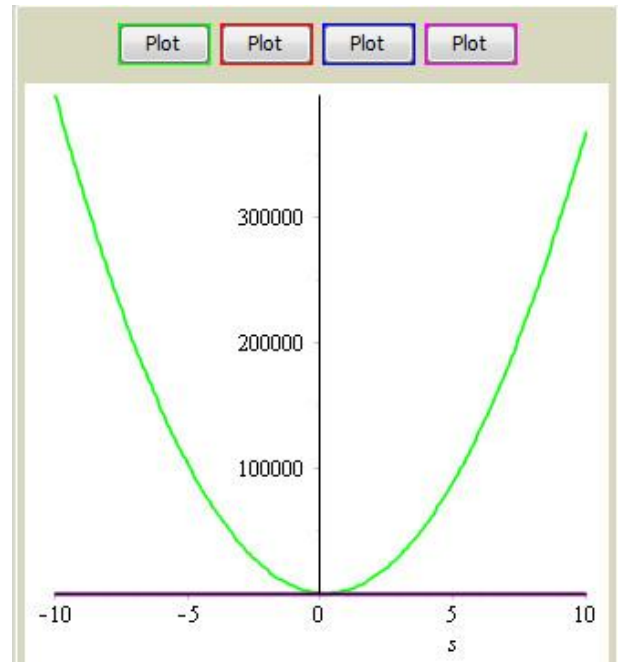


Fig. 7 .2-D graphing panel of the Maplet

Figure 7 is the 2-D graphing panel of the Maplet with “Plot” buttons. In this panel user can create 2-D plot of the solution of the given linear Fredholm integral equations. The buttons are in different colors. Each buttons is left for each solution methods.

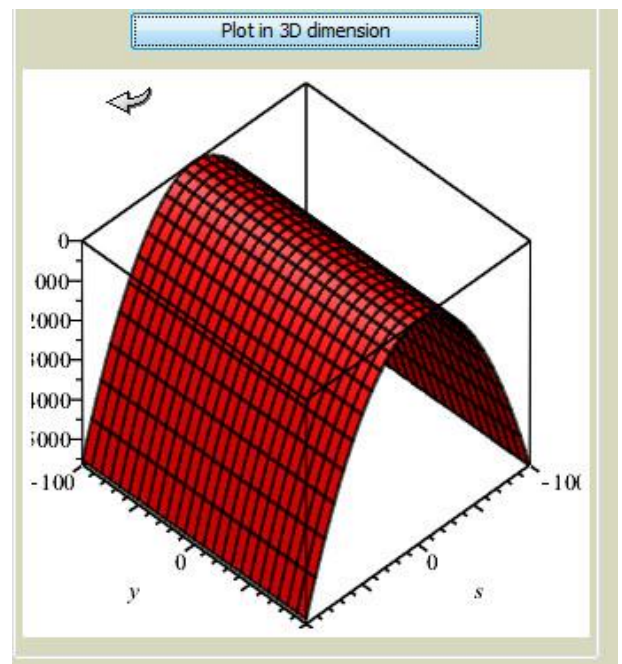


Fig. 8 .3-D graphing panel of the Maplet

Figure 8 is the 3-D graphing panel of the Maplet with “Plot in 3D” button. User can create the 3-D graph of the solution of the integral equation.

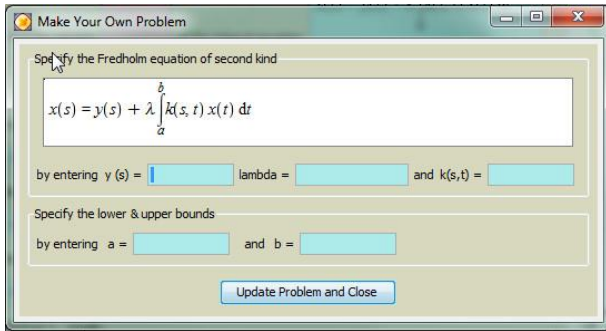


Fig. 9 .Make own problem panel of the Maplet

Figure 9 is the “Make own problem” panel of the Maplet. By this panel user can enter his/her own equation just by entering Kernel function, lambda, function y, the bounds a and b. After making these changings user push the “Update Your Own Problem” button.

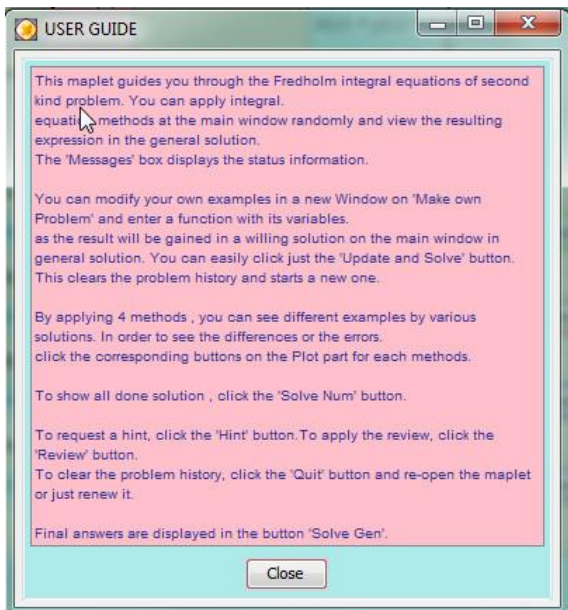


Fig. 10 .USER GUIDE panel of the application

Figure 10 is the “USER GUIDE” panel of the application. User can get general information about the use of the Maplet and Solution methods of linear Fredholm integral equations.

V. CONCLUSION

The incorporation of the Maplets as part of the lectures brings to the classroom the dynamic of three dimensional mathematical objects to create a more active way of teaching integral equations. The interfaces provide the instructor and student with simulations bringing integral equations concept alive. The interactive Maplets enable the user to control the graphics to improve their understanding of mathematical concepts.

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